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# 高速织机经纱振动特性分析*

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**摘要**:针对经纱振动影响织机织造性能的问题,对高速织机织造过程中经纱的横向与纵向耦合振动进行了研究,建立了经纱横向和纵向振动的耦合动力学模型,通过采用弹性力学方法和非线性振动方法对经纱运动的波动过程进行了分析,针对织造过程中的经纱黏弹性本构关系,选用了Kelvin模型,根据牛顿定律建立了经纱横向与纵向的振动微分方程。并利用Galerkin方法离散运动方程,分离时间和空间变量,将复杂的偏微分方程转化为常微分方程,采用四阶龙格库塔方法和Matlab,仿真分析了经纱横向振动和纵向振动特性以及经纱参数对经纱振动的影响。研究结果表明:在经纱织造过程中,对于小振幅的经纱耦合振动,经纱纵向振动几乎不影响横向振动,通过改变相应的经纱参数能够有效地降低经纱振动频率,有效地防止了经纱断纱,为经纱有效振动控制提供了理论基础。

**关键词**:高速织机;耦合;经纱;振动;仿真
**中图分类号**:TS101.2;TH133.1 **文献标志码**:A **文章编号**:1001-4551(2015)02-215-04

# Vibration analysis of warp yarns of high speed loom

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**Abstract**: Aiming at the problems of the weaving performance of loom affected by the vibration, the coupling vibration of warp yarns in lengthwise and crosswise were researched in the process of the weaving of high speed loom, the dynamics model of the coupling vibration of warp yarns in lengthwise and crosswise was established, the fluctuation process of warp yarn was analyzed by the nonlinear vibration method and elastic mechanics. Kelvin model was selected as warp viscoelastic constitutive relations. A differential equation of the warp yarn vibration in lengthwise and crosswise was established by Newton law. And the variables of time and space were separated by using Galerkin truncation method.The complex partial differential equations were converted into ordinary differential equations. The vibration characteristic of warp yarns in lengthwise and crosswise were analyzed by 4-order Runge-Kutta method and Matlab. The vibration of warp was affected by the parameters of warp. The results indicate that transverse vibration of warp is hardly affected by longitudinal vibration for small amplitude coupling vibration of warp in the process of the weaving of high speed loom. The vibration frequency of warp is effectively reduced by changing the corresponding parameters of the warp, and broken yarn is effectively prevented, the theoretical basis is provided for effective vibration control of warp yarns.

**Key words**: high speed loom; coupling; warp yarn; vibration; simulation

## 0 引 言

在实际工程中有许多轴向运动的材料,如人造纤维、纱线、运动带、履带和电梯缆绳等,在忽略弯曲应力时都可以简化成轴向运动弦线模型。1996年,Mote[1]首先研究了轴向运动弦线的非线性振动的问题。Wickert等[2]全面评论了1988年以前的相关研究工作。Pellicano等[3]和Chen[4]分别在2000年、2005年对轴向运动系统的研究工作进行了详细的介绍。其中,Riedel等[5]采用多尺度法研究了轴向运动系统的

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内部共振，Chen等[6]分析了粘弹性运梁的横向振动的稳态响应，丁虎等[7]对轴向变速运动黏弹性梁的受迫振动响应也作了一定分析。

粘弹性经纱的振动会引起经纱间的相互摩擦，影响经纱的张力与变形，降低经纱的断裂强度，对生产效率和织物质量有着重要的影响，而对经纱振动特性的研究也是为进行有效振动控制提供理论基础。

本研究针对轴向运动经纱系统的横向和纵向耦合振动问题，建立经纱横向和纵向耦合振动的动力学模型，分析耦合振动时纵向振动和横向振动。

## 1 经纱振动的数学模型

本研究采用kelvin模型[8]，则经纱的应力应变关系为：

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} = E^{*}\varepsilon(x,t) \tag{1}$$

式中：$E$ —经纱刚度系数，$\eta$ —动态粘性阻尼系数，$E^{*}$ —弹性模量。

引用Nayfeh非线性振动的结果，经纱在刚开始时有张力 $T_0$，在运动中经纱的长度开始发生变化，所以经纱内张力也在不断变化，张力的瞬时变化值[9]可表示为：

$$T = T_0 + \frac{E^{*}A(ds - dx)}{dx} = T_0 + E^{*}\varepsilon(x,t)A \tag{2}$$

假定线性阻尼力与经纱微段上一点的绝对速度成正比，设经纱的单位体积的阻尼系数为 $c$，则经纱微段所受阻尼力为 $cA\mathrm{d}x\mathrm{d}x/\mathrm{d}t$，$cA\mathrm{d}x\mathrm{d}y/\mathrm{d}t$，经纱微段的质量为 $dm = \rho A\mathrm{d}x$，纱线密度为 $\rho$，横截面积为 $A$，经纱微段受力如图1所示。

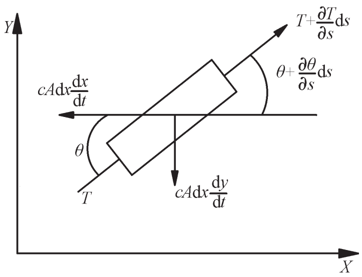



图1 经纱的微段受力图

笔者研究的是经纱的振动，在这个坐标系中研究点的纵向位移 $w(x,t)$，横向位移 $u(x,t)$。

运动方程为：

$$(T + \frac{\partial T}{\partial s}\mathrm{d}s)\cos(\theta + \frac{\partial \theta}{\partial s}\mathrm{d}s) - T\cos\theta - cA\mathrm{d}x\frac{\mathrm{d}x}{\mathrm{d}t} = \rho A dx\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} \tag{3}$$

$$(T + \frac{\partial T}{\partial s}\mathrm{d}s)\sin(\theta + \frac{\partial \theta}{\partial s}\mathrm{d}s) - T\sin\theta - cA\mathrm{d}x\frac{\mathrm{d}y}{\mathrm{d}t} = \rho A\mathrm{d}x\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} \tag{4}$$

对于中等挠度变形，通常将三角函数二阶Tayor展开[10]，即有：$\sin\theta \approx \theta$，$\cos\theta \approx 1 - \theta^2/2$，并把 $\mathrm{d}s = \mathrm{d}x/\cos\theta$，$\theta \approx \partial u/\partial x$，式(1,2)代入式(3,4)，消去高阶项，化简得到动力学方程：

$$\rho v^2\frac{\partial^2 w}{\partial x^2} + 2v\rho\frac{\partial^2 w}{\partial x\partial t} + \rho\frac{\partial^2 w}{\partial t^2} = \frac{\partial(E^{*}\varepsilon)}{\partial x} - (\frac{T_0}{A} + E^{*}\varepsilon)\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{1}{2}(\frac{\partial u}{\partial x})^2\frac{\partial(E^{*}\varepsilon)}{\partial x} - c(v + \frac{\partial w}{\partial x}v + \frac{\partial w}{\partial t}) \tag{5}$$

$$\rho\frac{\partial^2 u}{\partial t^2} + 2\rho v\frac{\partial^2 u}{\partial t\partial x} + (\rho v^2 - \frac{T_0}{A})\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial x}\frac{\partial(E^{*}\varepsilon)}{\partial x} + E^{*}\varepsilon\frac{\partial^2 u}{\partial x^2} - c(\frac{\partial u}{\partial x}v + \frac{\partial u}{\partial t}) \tag{6}$$

## 2 动力学方程的简化

本研究把kelvin型粘弹性本构关系 $E^{*}\varepsilon(x,t) = E\varepsilon + \eta\dot{\varepsilon}$ 及应变 $\varepsilon = \partial w/\partial x + (\partial u/\partial x)^2/2$ 代入式(5,6)。考虑到织机运行中，张力有一个周期变化的情况[11]，将 $T_0$ 换成 $T_0 + T_1\cos\Omega t$，并保留到三次非线性项，得到：

$$\rho v^2\frac{\partial^2 w}{\partial x^2} + 2v\rho\frac{\partial^2 w}{\partial x\partial t} + \rho\frac{\partial^2 w}{\partial t^2} = E\frac{\partial^2 w}{\partial x^2} + \eta\frac{\partial^3 w}{\partial x\partial x\partial t} + (E - \frac{T_0}{A} - \frac{T_1\cos\Omega t}{A})\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \eta\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x\partial t} + \eta\frac{\partial u}{\partial x}\frac{\partial^3 u}{\partial x\partial x\partial t} - E\frac{\partial w}{\partial x}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \eta\frac{\partial^2 w}{\partial x\partial t}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{1}{2}E\frac{\partial^2 w}{\partial x^2}(\frac{\partial u}{\partial x})^2 - \frac{1}{2}\eta\frac{\partial^3 w}{\partial x\partial x\partial t}(\frac{\partial u}{\partial x})^2 - c(v + v\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}) \tag{7}$$

$$\rho\frac{\partial^2 u}{\partial t^2} + 2\rho v\frac{\partial^2 u}{\partial x\partial t} + (\rho v^2 - \frac{T_0}{A} - \frac{T_1\cos\Omega t}{A})\frac{\partial^2 u}{\partial x^2} = \frac{3}{2}E(\frac{\partial u}{\partial x})^2\frac{\partial^2 u}{\partial x^2} + 2\eta\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x\partial t} + \eta(\frac{\partial u}{\partial x})^2\frac{\partial^3 u}{\partial x\partial x\partial t} + E\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} + \eta\frac{\partial u}{\partial x}\frac{\partial^3 w}{\partial x\partial x\partial t} + E\frac{\partial w}{\partial x}\frac{\partial^2 u}{\partial x^2} + \eta\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 w}{\partial x\partial t} - c(\frac{\partial u}{\partial x}v + \frac{\partial u}{\partial t}) \tag{8}$$

引入如下新的无量纲变量：

$$u^{*} = \frac{u}{L}, w^{*} = \frac{w}{L}, x^{*} = \frac{x}{L}, t^{*} = t\sqrt{\frac{T_0}{\rho AL^2}}, \gamma = v\sqrt{\frac{\rho A}{T_0}}, \lambda = \frac{T_1}{T_0}, \omega = \Omega\sqrt{\frac{\rho AL^2}{T_0}}, E_e = \frac{EA}{T_0}, E_v = \eta\sqrt{\frac{A}{\rho T_0 L^2}}, c_1 = \frac{ALcv}{T_0}, \mu = c\sqrt{\frac{A}{T_0\rho}} \tag{9}$$

将式(9)代入式(7,8)，化简，可得到无量纲化的动力学方程：

$$\gamma^2\frac{\partial^2 w}{\partial x^2} + 2\gamma\frac{\partial^2 w}{\partial x\partial t} + \frac{\partial^2 w}{\partial t^2} = E_e\frac{\partial^2 w}{\partial x^2} + E_v\frac{\partial^3 w}{\partial x\partial x\partial t} + (E_e - 1 - \lambda\cos\omega t)\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + E_v\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x\partial t} - E_v\frac{\partial u}{\partial x}\frac{\partial^3 u}{\partial x\partial x\partial t} - E_e\frac{\partial w}{\partial x}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - E_v\frac{\partial^2 w}{\partial x\partial t}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{1}{2}E_e\frac{\partial^2 w}{\partial x^2}(\frac{\partial u}{\partial x})^2 - \frac{1}{2}E_v\frac{\partial^3 w}{\partial x\partial x\partial t}(\frac{\partial u}{\partial x})^2 - c_1 - c_1\frac{\partial w}{\partial x} - \mu\frac{\partial w}{\partial t} \tag{10}$$

$$\frac{\partial^2 u}{\partial t^2}+2\gamma\frac{\partial^2 u}{\partial t\partial x}+(\gamma^2-1-\lambda\cos\omega t)\frac{\partial^2 u}{\partial x^2}=$$
$$E_e\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}+E_e\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}+E_v\frac{\partial u}{\partial x}\frac{\partial^3 w}{\partial x\partial x\partial t}+$$
$$E_v\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 w}{\partial x\partial t}+\frac{3}{2}E_e(\frac{\partial u}{\partial x})^2\frac{\partial^2 u}{\partial x^2}+2E_v\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial x\partial t}+ \tag{11}$$
$$E_v(\frac{\partial u}{\partial x})^2\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial t}-c_1\frac{\partial u}{\partial x}-\mu\frac{\partial u}{\partial t}$$

对于两端简支的边界条件为：

$$u(0,t)=u(1,t)=0,w(0,t)=w(1,t)=0 \tag{12}$$

# 3　动力学方程的离散化

本研究应用Galerkin[12]截断的方法对方程离散作近似处理,分析轴向运动经纱的纵向振动和横向振动,其振动的纵向和横向位移变量可表示为：

$$w(x,t)=\sum_{i=1}^{N}q_i(t)\sin(i\pi x) \tag{13}$$

$$u(x,t)=\sum_{i=1}^{M}b_i(t)\sin(i\pi x) \tag{14}$$

式中：$q_i(t),b_i(t)$（$i$=1,2,…）—经纱的广义坐标位移；$\phi_i(t)$（$i$=1,2,…）—满足边界条件且具有正交性的试函数。本研究利用三角函数基的正交性,取$N=M=2$,得到常微分方程组：

$$q''_1-\frac{16}{3}\gamma q'_2+\pi^2E_vq'_1+(\pi^2E_e-\pi^2\gamma^2)q_1+$$
$$\pi^3b_1b_2(E_e-1-\lambda\cos\omega t)+\pi^3E_vb_1b'_2+\pi^3E_vb_2b'_1-$$
$$\frac{3}{8}\pi^4E_vb_1^2q'_1-\frac{1}{8}\pi^4E_eb_1^2q_1-2\pi^4E_eq_2b_1b_2-2\pi^4E_vq'_2b_1b_2-$$
$$\pi^4E_eb_2^2q_1-\pi^4E_vb_2^2q'_1+\frac{4}{\pi}c_1-\frac{8}{3}c_1q_2+\mu q'_1=0 \tag{15}$$

$$q''_2+\frac{16}{3}\gamma q'_1+4\pi^2q_2(E_e-\gamma^2)+(4\pi^2E_v+\mu)q'_2+$$
$$\frac{1}{2}\pi^3b_1^2(E_e-1-\lambda\cos\omega t)+\pi^3b_1b'_1E_v-2\pi^4E_eq_1b_1b_2-$$
$$6\pi^4E_eq_2b_2^2-2\pi^4E_vb_1b_2q'_1-\pi^4E_eb_1^2q_2-$$
$$6\pi^4b_2^2q'_2E_v-\pi^4E_vb_1^2q'_2+\frac{8}{3}c_1q_1=0 \tag{16}$$

$$b''_1-\frac{16}{3}\gamma b'_2-\pi^2(\gamma^2-1-\lambda\cos\omega t)b_1+\pi^3E_eq_1b_2+\pi^3E_eq_2b_1+$$
$$\pi^3E_vb_2q'_1+\pi^3E_vb_1q'_2+\frac{3}{8}\pi^4E_eb_1^3+3\pi^4E_eb_1b_2^2+$$
$$\frac{3}{4}\pi^4E_vb_1^2b'_1+4\pi^4E_vb_1b_2b'_2+2\pi^4E_vb_2^2b'_1-\frac{8}{3}c_1b_2+\mu b'_1=0 \tag{17}$$

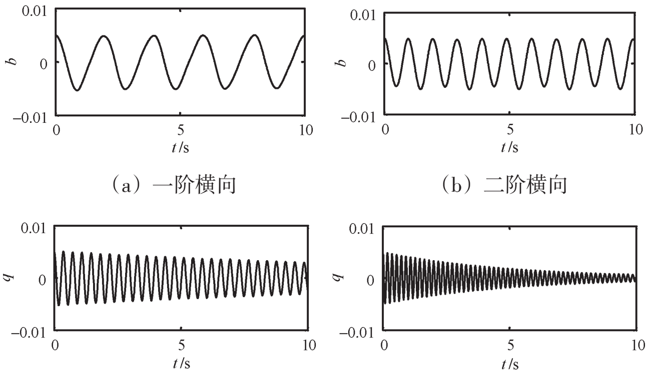
$$b''_2+\frac{16}{3}\gamma b'_1-4\pi^2b_2(\gamma^2-1-\lambda\cos\omega t)+\pi^3b_1q_1E_e+$$
$$\pi^3b_1q'_1E_v+(3E_e+2E_v)\pi^4b_1^2b_2+2\pi^4b_2^3(3E_e+2E_v)+ \tag{18}$$
$$4\pi^4E_vb_1b'_1b_2+8\pi^4E_vb_2^2b'_2+\frac{8}{3}c_1b_1+\mu b'_2=0$$

# 4　实验及结果分析

## 4.1　仿真实验

本研究应用四阶Runge-Kutta方法编写程序[13],并利用Matlab软件得到仿真实验结果。

根据纱线具体参数,设置运动经纱的几何参数为：$\gamma=0.1,E_e=30,c_1=0.02,\lambda=0.05,\omega=0.1\pi,E_v=0.01,\mu=0.01$,初始条件$q_1=0.005,q'_1=0,q_2=0.005,q'_2=0,b_1=0.005,b'_1=0,b_2=0.005,b'_2=0$,仿真计算时间设定$t$=10 s。运动经纱的一阶横向振动和二阶横向振动响应曲线如图2(a)、2(b)所示。运动经纱的一阶纵向振动和二阶纵向振动如图2(c)、2(d)所示。




（a）一阶横向　（b）二阶横向

（c）一阶纵向　（d）二阶纵向

图2　振动响应曲线

从图2中可以看出,轴向运动经纱的横向振动的响应远大于纵向振动的响应,所以经纱振动系统响应以横向振动为主导,从而得出：在高速织造时,经纱纵向振动的影响不会改变经纱横向振动非线性的特性,一般在考虑经纱弱非线性或小振幅经纱振动时,可以对经纱振动系统的横向非线性进行一定的简化,忽略经纱纵向振动的影响,以方便对经纱振动的研究。

## 4.2　各参数对横向振动影响的数值分析

本研究通过对横向振动各参数的数值分析,确定哪些参数影响经纱的的横向振动,以便采取相应的措施,降低或者消除这些因素的影响。

纱线速度对经纱横向振动的影响如图3、图4所示。设定运动经纱的几何参数为：$E_e=0.2,c=0.01,\lambda=0.05,\omega=0.1\pi,E_v=0$,初始条件：$b_1=0.005,b'_1=0,b_2=0.005,$

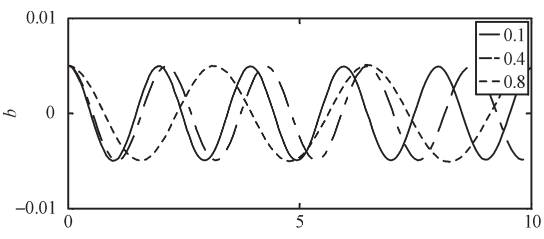



图3　纱线速度对一阶振动方程的影响比较图

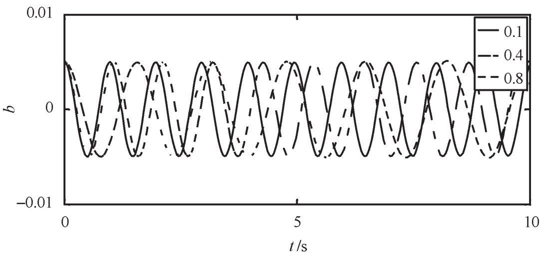



图4 纱线速度对二阶振动方程的影响比较图

$b'_2=0$ ,仿真计算时间设定 $t$ =10 s,则由图3、图4分析对比可得,当纱线速度分别取0.1,0.4,0.8时,可以看出随着纱线速度的增大,系统的振幅是不变的,但系统的频率是在减小。

张力波动比对经纱横向振动的影响如图5、图6所示。设定运动经纱的几何参数为: $\gamma=0.1,E_e=0.2,$ , $c=0.01,\omega=0.1\pi,E_v=0$ 初始条件: $b_1=0.005,b'_1=0,$ $b_2=0.005,b'_2=0$ ,仿真计算时间设定 $t$ =10 s,则由图5和图6分析对比可得,当张力波动比分别取0.05,0.2,0.4时,可以看出随着张力波动比的增大,系统的振幅和频率都在增大。

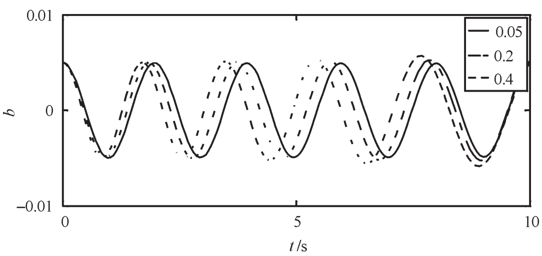



图5 张力波动比对一阶振动方程的影响比较图

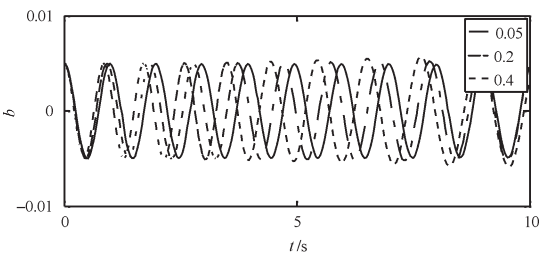



图6 张力波动比对二阶振动方程的影响比较图

## 5 结束语

本研究通过对经纱运动的动力学和数学建模,同时考虑材料和几何变形的非线性因素以及对振动耦合方程进行非线性振动分析和数值仿真分析,得到如下结论:

(1) 在高速织造时,由于经纱本身材料的特性以及几何变形非线性因素的影响,经纱可能发生的横向和纵向振动,都属于经纱非线性振动。

(2) 一般在考虑经纱弱非线性或小振幅振动时,可以对经纱振动系统的横向非线性进行一定程度地简化,忽略经纱纵向振动的影响。

(3) 通过对经纱各参数仿真分析,得到降低经纱横向振动频率的方法有提高纱线速度,减小经纱张力波动比。

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